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Examining Technology Uses in the Classroom: Developing Fraction Sense Using Virtual Manipulative Concept Tutorials

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Abstract

This paper describes a classroom teaching experiment conducted in three fifth-grade mathematics classrooms with students of different achievement levels. Virtual fraction manipulative concept tutorials were used in three one-hour class sessions to investigate the learning characteristics afforded by these technology tools. The virtual fraction manipulative concept tutorials exhibited the following learning characteristics that supported students during their learning of equivalence and fraction addition: (1) Allowed discovery learning through experimentation and hypothesis testing; (2) Encouraged students to see mathematical relationships; (3) Connected iconic and symbolic modes of representation explicitly; and (4) Prevented common error patterns in fraction addition.

Technology can play a major role in making sense of mathematics, and has been used in classrooms to enhance mathematics instruction. Students who use appropriate technology persist longer, enjoy learning more, and demonstrate gains in mathematics performance (Goldman & Pellegrino, 1987; Okolo, Bahr, & Reith, 1993). Recent developments in computer technology have created innovative technology tools available at no cost on the World Wide Web called *virtual manipulatives*. A *virtual manipulative* is "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002).

In this project, we used two virtual manipulative applets from the National Library of Virtual Manipulatives (http://matti.usu.edu/nlvm/index.html) and one applet from the NCTM electronic standards (http://nctm.org) to reinforce fraction concepts in three fifth-grade classes with students of different ability levels. Our goal was to examine learning characteristics that supported students during their learning of equivalence and fraction addition. The fraction applets used in this three-day project are *concept tutorials* with instructions on using the manipulatives and activities that accompany the applets. They provide links to the NCTM Standards (National Council of Teachers of Mathematics, 2000) and are beneficial for visual learners. Advantages of these sites are that they are interactive, give the user control and ability to manipulate objects, and provide opportunities to explore and discover mathematical principles (Cannon, Heal, & Wellman, 2000).

The use of multiple representations and the ability to translate among representational models has been shown to be an important factor in students' abilities to model and understand mathematical constructs (Cifarelli, 1998; Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Kamii, Kirkland, & Lewis, 2001; Lamon, 2001; Perry & Atkins, 2002). In a summary of over 100 research studies, Marzano (1998; see also Marzano, Gaddy, & Dean, 2000) found that one instructional techniques that demonstrated a consistent positive impact on student achievement was the use of graphic/nonlinguistic formats to explore and practice new knowledge. The virtual manipulative concept tutorials used here have the following characteristics – they are dynamic computer-based objects that can be manipulated, and they are presented in a fluid graphic/nonlinguistic format. The Lesh Translation Model also highlights the importance of students' abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols (Lesh, Cramer, Doerr, Post, Zawojewski, 2003). Combining various modes of representation is a feature of the virtual manipulative concept tutorials used during these lessons with these students. This framework guided our development of the tasks and materials for student learning in this project.

The teachers chose the topic of fractions for this project because of the challenge it often poses for students. Students have less out-of-school experiences with fractions than with whole numbers, making it necessary for teachers to provide relevant experiences to enhance students' informal understanding of fractions and help connect procedural knowledge with conceptual understanding (National Research Council, 2001). Developing visual models for fractions is critical in building understanding for fraction computation. Yet conventional instruction on fraction computation tends to be rule based. We based this project on the early results of other classroom research involving virtual fraction manipulatives where researchers have reported positive results on student achievement (Moyer, Suh, & Heo, 2004), confidence (Heo, Suh, & Moyer, 2004) and aesthetic attributes, like ease of use and enhanced enjoyment for users (Reimer & Moyer, in press). This project explored ways in which virtual manipulatives could facilitate the connection between conceptual and procedural understanding by capitalizing on the graphic/nonlinguistic features of the virtual fraction manipulatives.

Methods

Many teachers are not aware of the capabilities of virtual manipulatives and do not currently use them in lessons during regular mathematics instruction. We found it challenging to find resources and research on which to base the design of this project. For this reason, we chose to create teacher-made handouts and assessments to guide and evaluate the students and their use of several virtual fraction manipulatives in this exploratory classroom project. We were particularly interested in answering the following question: (1) What learning characteristics are afforded by the use of the virtual fraction manipulatives for understanding fraction equivalence and addition?

Participants

The participants in this study were 46 fifth-grade students in three classes at the same elementary school. The school identified the three student groups as Low, Average, and High achievement based on standardized testing results used at the school. There were 21 students in the High achievement group, 12 in the Average achievement group, and 13 in the Low achievement group. The classes were made up of a diverse student population with 27% minority and an average socioeconomic base.

Procedures

The entire project was conducted in the elementary school's computer lab where students interacted with virtual manipulatives on three days for one hour each day. There was a large screen where the teacher could display electronic examples for the whole class to view in the lab and 25 computers, allowing all students to have their own computer. Students worked independently during the virtual manipulative lessons and participated in the project during their regularly scheduled mathematics class sessions.

The first author of this article, an elementary-school teacher, taught a unit on fractions to the three groups of students to reduce the impact of any teacher effects. She led instruction and discussions with the students during all of the class sessions. The teacher was familiar with all of the students and taught these students the previous year. The teacher used the same task sheets and virtual manipulatives with each of the three groups. The fifth-grade state standards were used as guides for the mathematics content of the lessons. The fraction standards addressed during the lessons included fraction equivalence and addition of fractions with unlike denominators.

Each lesson began with an introduction to the virtual manipulative applet that would be used that day and several mathematical tasks for the students. On each day in the computer lab, students received a teacher-made task sheet that provided instructions for using the virtual manipulatives, several problems, and space to record work. These directions focused students on the mathematical tasks during the lessons. The teacher reviewed the instructions with the class and modeled how to use the virtual manipulatives before students worked independently on the activities.

Data Sources

Several sources of data were collected during the project to answer the research question including observation field notes, student interviews, and classroom videotapes. Interviewers and one researcher with a video camera were present in the classroom during these sessions. Three interviewers interacted with students while the students were interacting with the virtual manipulatives. Every student was interviewed at least twice on each of the three days. Different interviewers spoke with different students on different days of the project. The interviewers used fieldnotes and a video camera to record the data. This allowed analysis of complete responses from different students with different interviewers. The interviewers asked two to four questions of each student during these interactions. When students worked on finding equivalent fractions, they were asked questions such as: (1) How do you find an equivalent fraction using these fraction pieces, and (2) Is there a pattern in the list of equivalent fractions? When students worked on addition of fractions, they were asked questions such as: (1) Can you model and explain how you would add these fractions using the manipulatives? and (2) Can you explain to me how to add when you have different denominators? These conversations were transcribed so that a written record of students' direct quotations during their work was recorded. We analyzed these comments using a narrative analysis procedure to identify dominant themes in students' experiences (Shank, 2002).

Results

The results below focus on the observations, student interviews, and videotapes of students' interactions with the virtual manipulatives during the project to determine the learning characteristics of the technology that may have supported student understanding.

Task 1: Finding Equivalent Fractions

During the first task, the teacher demonstrated the Renaming Fractions applet showing students how 1/2 could be renamed to 2/4 and 3/6 using the pictorial representations provided by the virtual fraction manipulative (Available at the National Library of Virtual Manipulatives, matti.usu.edu/nlvm/index.html). The focus of this introduction was on the visual representation of the two fraction models. This particular applet allowed students to manipulate an up and down arrow key that divided each region into multiple parts. By using the arrow keys, students could find equivalent fractions. Students were assigned the task of finding more than one equivalent fraction for each fraction given by the teacher. (See fig. 1.) The teacher asked students to work on five problems and to record their work on a recording sheet. Students wrote down a list of equivalent fractions such as 1/2=2/4=3/6=4/8. They were also asked to record one equivalent fraction on a class-recording poster. While they worked, students looked for patterns and created their own rules for finding equivalent fractions.

At the beginning of the class session, students clicked on the arrow key slowly until the lines evenly divided the region. As they became more rehearsed in the activity, they seemed to click quickly until they reached a common multiple. Students also appeared to use their knowledge of multiples to anticipate when to stop clicking.

During the observations and interviews, many students used the virtual manipulatives to find consecutive equivalent fractions like 1/4, 2/8, 3/12 and 4/16. However, other students listed a random sequence of equivalent fractions such as 1/4, 5/20 and 12/48. Some of the students experimented with the arrow keys to find an equivalent fraction with the greatest number for the numerator and denominator. Once they realized the highest number on the virtual applet for the denominator was 99, they clicked the down arrow to find the closest multiple for that fraction (ex. 1/8=12/96).

Students were asked questions such as, "How does the equivalent button help you?" One student replied, "When you click on the arrow button, you look at the black lines and the blue lines, and the lines have to be on top of each other (line up) and match. That's how you know they are equivalent. Then you can count the pieces." Another student was asked, "What patterns do you notice when you change the fraction names?" The student replied, "You get more pieces, but it's still the same amount. Each piece gets thinner."

During the discussion at the end of the class session students sat together in front of the poster of equivalent fractions and explained the ways they used the virtual manipulative applet for finding equivalent fractions. Students defended and challenged some of their classmates' rules for finding equivalent fractions. One student found a pattern of “repeated addition of numerators and denominators,” while another with the same idea named it as “finding all the multiples for the numerator and the multiples for the denominator.” Some students used multiplicative reasoning stating they were “doubling or tripling the numerator and denominator” while another student described the same act as “multiplying by the same number for the top and bottom number.” The students demonstrated multiple ways of thinking about the equivalent fractions.

We prepared a narrative summary of the observation and interview data to identify important themes that emerged during the firsts task. Observations and student comments indicated that the Renaming Fractions applet supported student learning in three important areas: (1) It allowed discovery learning, (2) It allowed students to make conjectures, and (3) It encouraged students to see mathematical relationships.

Discovery learning. This fraction applet features an arrow button that enables students to experiment with various visual representations of the denominator up to 99. When using this applet, students are able to view multiple visual images of different fractions very quickly. They can see what each fraction looks like in both its symbolic and pictorial modes in the on-screen presentation. The connection of these visual and symbolic models through experimentation allowed students to rapidly view many different fractions from halves to 99ths.

Making conjectures. While using the Renaming Fraction applet, students were able to test what would happen to the visual images of the fraction pieces as they pressed the arrow keys up and down. This allowed them to test their hypotheses about fractions they thought might be equivalent to a given fraction. As they viewed numerous different visual images of the fractions, the applet may have facilitated their ability to look for and identify patterns in the fractions. This feature may have led them to a conjecture about a working rule for the patterns they were seeing.

Mathematical relationships. Students observed that the lines on the fraction applet that divided the whole region would not line up evenly when the pieces in the region were not divided by a common multiple. Each time students viewed a new fraction, the applet reinforced this mathematical principle. After working at their own pace with numerous fraction examples provided by the applet, students began to see mathematical relationships related to the factors and multiples of the numbers they were given.

Task 2: Adding Fractions with Unlike Denominators

During the second task, the teacher introduced the Fraction Addition applet on the National Library of Virtual Manipulatives website (matti.usu.edu/nlvm/index.html). The applet gave two fractions with unlike denominators and linked a pictorial representation of the fraction addition process with a symbolic representation of the fraction exercise.

Students were asked to rename the fractions so that they had common denominators. Similar to the Renaming Fractions applet, the Fraction Addition applet allowed students to use the arrow button to search for a fraction denominator that was common to both fraction pieces. (See fig. 2.) Visually, students could see how the total number of pieces changed on each of the fractions as they used the arrow button.

During these tasks, students clicked on each fraction until they found a number that could be used to divide both of the fractions into equal parts. Once students found a common denominator, they renamed the fractions. When they clicked on a “Check” button, they were given immediate feedback, and the screen changed to the addition mode. (See fig. 3.) Students were then able to add the fractions that they had renamed with common denominators. The applet enables students to drag the fraction pieces from each of the fraction addends into a region for the sum (typically the region was a square or a circle). Then students counted the total number of fraction pieces in the region and typed their sum in the answer box.

The observers noted that students used the visual model to work through the problems. However, students in the Highest-level group were much more efficient at this process. For students in the Average and Low achievement groups, the visual model appeared to act as a scaffolding mechanism providing assistance in finding common denominators for those who needed it. Many of the students in these two groups were much more methodical and worked through the concept tutorial step by step, working with the visual model first, and then typing in the numbers on the fraction addition statement. One of the students in the low achievement group commented, “I like the virtual fractions because its better than writing it down. The computer helps you find the multiples when you click the arrow key until you get equal pieces.” When an interviewer asked one student what she thought about using the virtual manipulatives to learn more about fractions, she replied, “I like when you move the two fraction pieces to the whole piece and put them together. That's pretty fun,” and another student commented, “It let me see the fraction problem visually.” The students’ written work also indicated that using the virtual fraction applets gave them the opportunity to attach meaning to fraction symbols while building rational number sense.

One characteristic observed in the processing of the Highest-level group was common among students who already had a good grasp of the algorithm for adding fractions from prior instruction. These students typed in the numbers for numerators and denominators first. Then they checked their answers by manipulating the pictorial models of the two fraction addends. For example, students in the highest-level group were observed using paper and pencil to create a list of common multiples, and then they used this list to enter the common denominator on the applet and check their calculations. The “Check” button was helpful for all students because it gave them immediate feedback. One student commented on using the check button on the applets: “I like it better than writing it down because it tells if you are wrong and tells you what you've done wrong so you can fix it.”

When the teachers asked students, "How does the visual model on the virtual fraction help you," students responded with comments that indicated facility in understanding due to the visual and dynamic nature of the program. For example, one student commented, “It helps me to see pictures of what they look like instead of just working with numbers,” while another stated, “It helps me so I can find the same denominator by seeing if the bars are the same size. I count the pieces that are shaded and use them as the numerator and then I add." One of the students explained, "When it's in your head, you might forget it, but when you use the computer pictures, it's right there and it stays there." This link between the visual model and the mathematical symbols provided support for many of the students.

Students stated specifically that they liked the step-by-step process, which showed visually what was happening to the two fraction addends during the addition process. As one student commented, "It actually shows step-by-step how the number changes to the fraction," while another student reported, "I like the way the computer tool went step-by-step." The visual images seemed to be a key to helping several students: "It's easier to use the pictures and I like moving the fraction blocks. I like when you take two different fraction pieces and move it to the whole block to put them together. It made it easier for me to add."

We prepared a narrative summary of the observation and interview data to identify important themes that emerged during the second task. Observations and student comments indicated that the Fraction Addition applet supported student learning in two important areas: (1) It linked symbolic and iconic modes, and (2) It helped to deter a common fraction error pattern.

Linking symbolic and iconic modes. On this fraction applet, the symbolic and iconic manipulations are closely tied together during each step of the process. Students frequently have difficulty associating their manipulations with fraction pieces to the symbolic process of using algorithms. Although they may be successful manipulating physical fraction pieces, they may be unsuccessful at manipulating numeric representations because they see these as two separate processes. The addition of fractions using the virtual fraction manipulatives connected the processes students were using in finding common denominators and combining fraction pieces with the symbolic representation of these concepts. The first screen of the applet shows two fraction pieces with unlike denominators, which students rename using arrow keys. Students type in the numeric representation of the renamed fractions, and the screen changes to allow the student to perform the addition in both symbolic and iconic forms. After students type the numeric representation into the addition sentence, they receive feedback on the accuracy of their response.

Preventing a common error pattern. One common problem in the addition of fractions with unlike denominators is known as the "add across" error where students add both the numerators and the denominators (Ashlock, 2002) (i.e., 1/3 +2/6= 3/9). Using this fraction applet, students are guided through the problem with the support of a visual model. In each problem, the concept tutorial does not allow students to add the numbers erroneously and waits until the two fractions have a common denominator. Only after students find common denominators are they permitted to begin the addition process. Within the addition mode, the ability to drag the shaded fraction pieces to the sum square/sum circle further reinforces the idea of combining like pieces to determine the numerator. Another feature is that the sum square/sum circle names the total divided parts of the denominator. Students using the concept tutorial to practice addition of fractions cannot make this common mistake because of the step-by-step process that includes both a visual and symbolic representation. It prevents this common error by allowing students to reason through fraction numbers while building on their strategic competence.

Task 3: Fraction Track Game

The Fraction Track Game was a culminating activity for Task 3. This fraction applet requires students to apply concepts of fraction equivalence and addition. (See fig. 4.) Students used NCTM's e-resources for the fraction track game (www.nctm.org). The teacher explained the rules for the game and students played the game in pairs. The object of the game is to use fractions given by the applet to move markers along fraction tracks that range from the zero to one. The seven fractions tracks are marked off in the following increments: ½, ⅓, ¼, □, □, ⅛ , and □10.

During this task, we were particularly interested in whether or not students would connect their experiences with Tasks 1 and 2 and apply these skills to the Fraction Tracks Game. The observations indicated that some of the students who were successful at finding equivalent fractions were also able to apply those strategies to move markers along the fraction tracks. For example, when one student was given the fraction ¼ and he already had a marker on the track for fourths, he quickly moved the marker on the eighths track to the fraction 2/8. Students who had difficulty renaming the fractions were observed passing their turn when this situation arose, because they did not realize, for example, that 2/8 is equivalent to ¼.

Students verbalized their thinking more frequently during the third task because they were working with a partner. Although the students may not have realized it during the class session, the observers noted that students were making comments that showed they were using fraction addition and subtraction processes. For example, students made comments such as, “I just need ¼ to win,” and “I can win with a 2/8 to get to one whole.” These statements demonstrated that students were thinking about the residual or the “left over part” when they added fractions. Similar findings were reported in the Rational Number Project, where students who used fraction circles for an extended period of time were able to compare fractions by using mental images in thinking about the residual (Cramer, Behr, Post, & Lesh, 1997).

The following questions were used by the interviewers during Task 3: (1) How are the various tracks on the board related? (2) How does equivalence help you play fraction tracks? (3) How does addition of fractions help you play the game? (4) What strategies help you win?

Students’ initial responses to these questions at the beginning of the task showed little knowledge of any relationships among the tracks on the fraction track board. One student describing the Fraction Tracks Board said, “It's even, odd, even.” Another student said, “The fraction number increases.” However, as students played the game and became more familiar with the board, many students discovered that the board showed equivalent fractions in a vertical alignment. One student responded, “Some fractions are equivalent.” Another student used his finger to point and make a vertical line on the screen. He followed the fractions from the top to the bottom of the screen vertically, beginning at ½ and said, “One-half, two-fourths, three-sixths, and four-eighths are equal.” Initially, most students did not use an equivalent fraction to move their markers ahead. As they continued to play the game, some of them realized that using an equivalent fraction would allow their markers on the game board to reach the end goal faster. One student commented about his strategy: “I got the fraction □, so I multiplied it to get 4/10 to move the marker on the tenths track.” As you can see from this student’s comment, although he is able to find an equivalent fraction for □, he does not recognize that this relationship is presented on the Fraction Tracks game board.

Two girls were asked if addition helped them play the game. The students looked at the interviewer blankly and stated that they had not used any addition in the game. The observer continued watching this pair as they played the game. One of the girls had a marker on the track for sixths at 4/6 and the computer gave her the fraction 2/6. She quickly moved her marker to the end goal, saying, "I win." Then she turned to the observer and said, “I just added! 4/6 + 2/6 is 6/6 or 1 whole.”

We prepared a narrative summary of the observation and interview data to identify important themes that emerged during the second task. Observations and student comments indicated that the Fraction Tracks Game produced two additional themes that were not part of the first two tasks: (1) student-to-student communication of mathematical ideas and (2) the application of previously learned concepts to a different mathematical situation.

Student-to-student communication. The Fraction Tracks Game is designed for students to play the game in pairs. By allowing the students to work in pairs, students had the opportunity to talk about the mathematics involved in the game. The virtual environment served to facilitate students’ interactions with each other and with the computer. When one student made a move on the Fraction Tracks game board, the student’s partner occasionally questioned the student’s move. This led students to justify their choice of move and to explain why the move was mathematically accurate. Much of the mathematics talk during the class session focused on students’ agreement on whether or not a move on the Fraction Tracks board was legal mathematically.

Application of previously learned skills. Students used the skills of renaming fractions, finding common denominators, and fraction addition to play the Fraction Tracks Game. The students learned and reinforced these concepts by playing the game because it allowed them to rehearse and play several times to develop their strategies. During this rehearsal they began to recognize that they could use previously learned skills to improve their performance during the game and so they applied these skills. This enabled many of the students to attach meaning to equivalence and addition of fractions. However, although it was evident to the observers, it was not immediately obvious to students that they were applying previously learned skills to the new mathematical situation.

Discussion

This paper points to some of the characteristics of virtual manipulative tutorials that may be beneficial for students as they are learning mathematics concepts. One characteristic afforded by the virtual manipulative concept tutorials used in this project was their design that combined both visual and symbolic images in a linked format. This may have encouraged students to make connections between these modes of representation and, thereby, developed students’ representational fluency, particularly for visual learners. In our observations and analyses, the students identified as Low achievement seemed to benefit most out of the three groups of students from working with the virtual concept tutorials.

Another characteristic of these virtual fraction manipulatives was that the applets allowed students to experiment and test hypotheses in a safe environment. The guided-format features of the applets allowed guessing and trial-and-error, and at the same time, would not allow students to submit an incorrect response. The applets are designed to provide students with opportunities to see patterns in the mathematical processes they are doing. For example, students could view two equivalent fractions in a visual format as well as their related symbolic notations. They could also model the process of moving two addends to create a sum to see that the addends became parts of the same whole, and how to represent that sum symbolically. Students had the opportunity to try out an answer and be incorrect without receiving judgmental feedback on their error. This understanding may have helped to discourage common fraction error patterns.

Virtual manipulative technology is a promising tool for improving students' visual and conceptual abilities in mathematics. The dynamic nature, along with color, graphics, and interactivity can capture and hold the attention of students so that they persist in mathematics tasks. However, the challenge of using technology is similar to the challenges associated with using physical manipulatives – lack of knowledge on how to effectively use tools during mathematics lessons. One advantage of the virtual fraction manipulatives used in this project was that they were formatted as concept tutorials with teaching prompts embedded into the virtual applets. However, it was still important for the teachers to ask students questions individually and in a whole group culminating discussion to fully optimize students' learning experiences. This interaction provided opportunities for students to make sense of the mathematical concepts they were exploring.

Virtual manipulative technologies are free and available to schools and teachers on the World Wide Web through an Internet connection. However, many schools still lack the resources necessary to keep computers in service or to maintain connections to the Internet. Unfortunately adequate computer labs with reliable Internet connections and technical support to maintain them are not available in numerous schools throughout the country. These factors inhibit teachers' and students' opportunities to use these tools for mathematics teaching and learning.

Another important, though not observable, characteristic of these technologies is that virtual manipulative representations may model the fluid nature of thinking. Our cognitive processes when working with mathematical ideas use a variety of trial and error, planning, experimenting, and visualizing constructions. By allowing students to manipulative on-screen objects to test hypotheses and experiment with ideas, the virtual manipulatives may more closely model the dynamic nature of our thinking. These on-screen objects can also be preserved by saving and printing, thus showing a record of a student's thinking when working on a particular problem or mathematical situation. A medium for students to experiment with mathematics in a more fluid and dynamic way may enhance students' thinking and creativity. For example, in a project using virtual pattern blocks in a kindergarten classroom to create repeating and growing patterns, researchers found that the children used more elements in their pattern stems, created a greater number of patterns, and exhibited twice as many creative behaviors when using the virtual blocks as they did using wooden pattern blocks or making drawings (Moyer &

Niezgoda, 2003). Today's students have increasing facility with various forms of technological tools outside the school classroom. Technological tools that facilitate student thinking during mathematics lessons, such as virtual manipulatives, have the potential to provide students with different kinds of mathematical experiences.

Technology is a powerful tool; yet in the education arena, it has not reached its potential as an instructional tool. Teachers may not embrace technology because they believe the Internet is unreliable or that a particular web site did not prove to be worthwhile, or they may be limited by the lack of technology resources and support. We hope that this project provides insights into instructionally powerful tools available for school mathematics, and that it encourages teachers to experiment with virtual manipulative technologies. We believe that teachers, researchers, and education technology developers can share their ideas and expertise to ensure that effective computer programs and applets will continue to be developed for mathematics teaching and learning.

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<u>Figure 1</u>. Renaming Fractions.

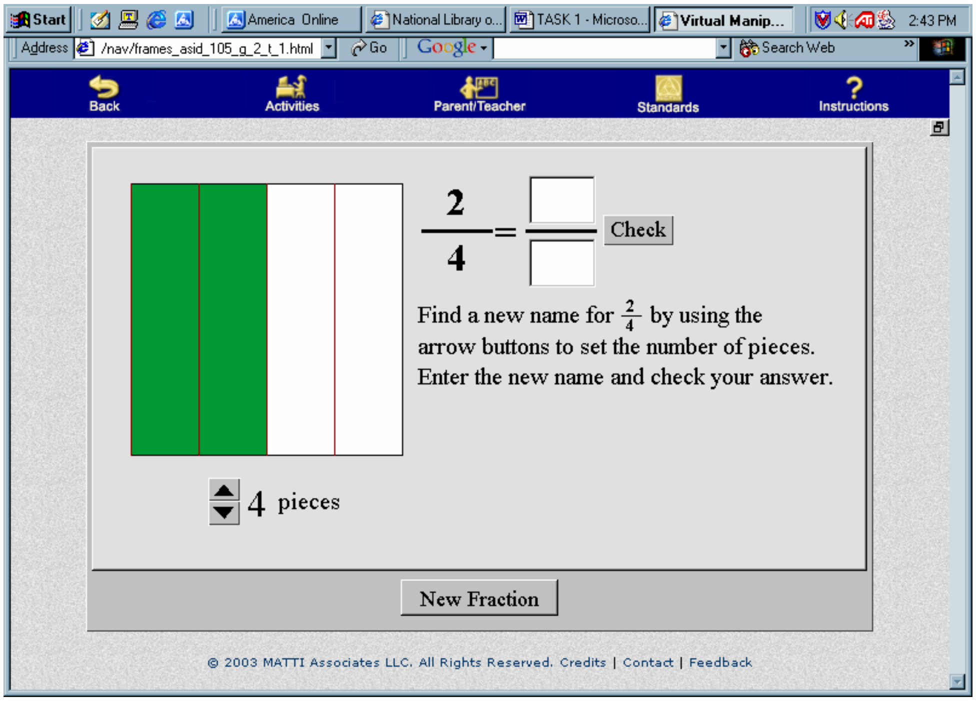


Figure 2. Fraction Addition –Finding Common Denominators.

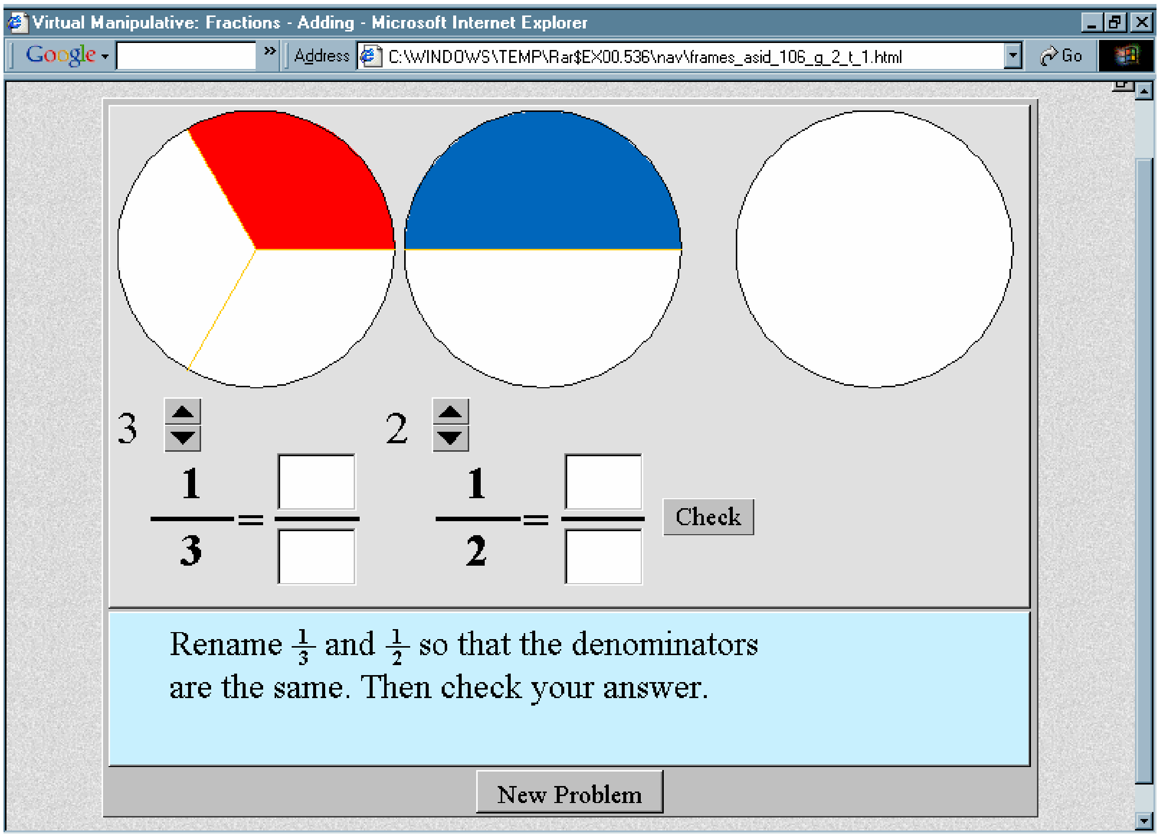


Figure 3. Fraction Addition.

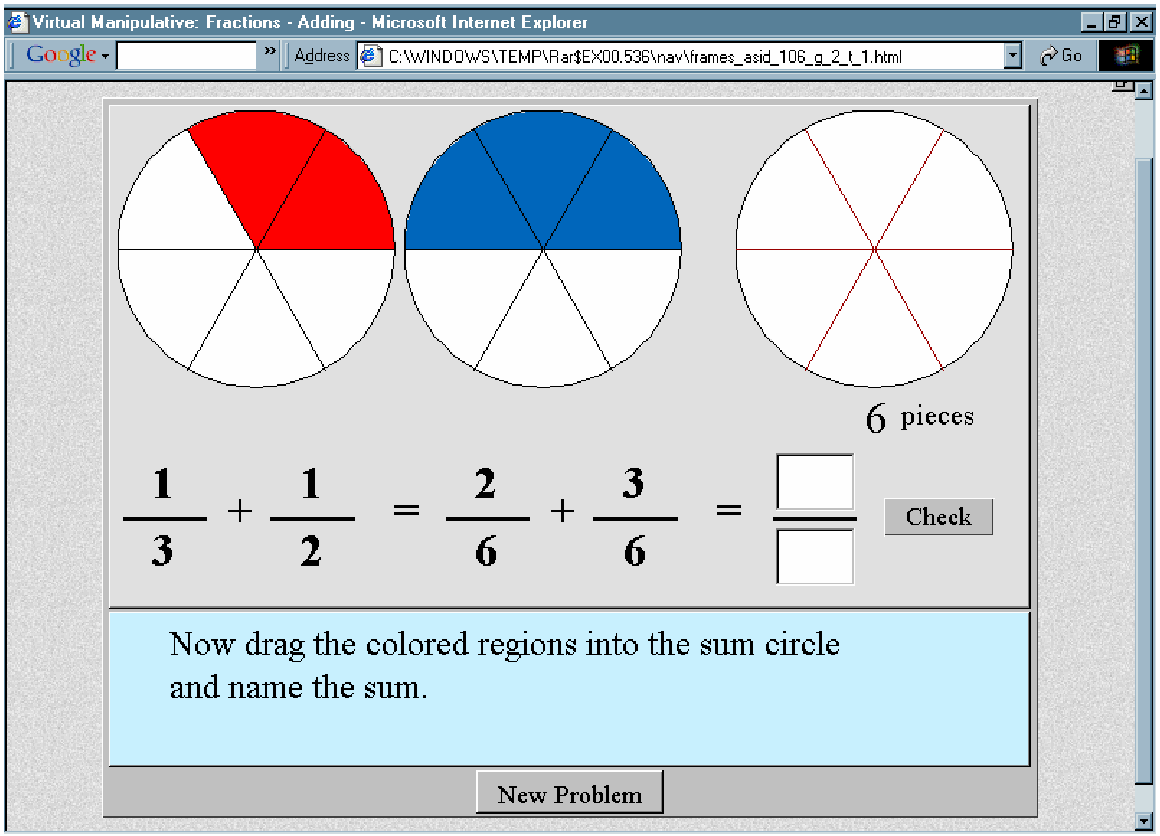


Figure 4. Fraction Track Game.

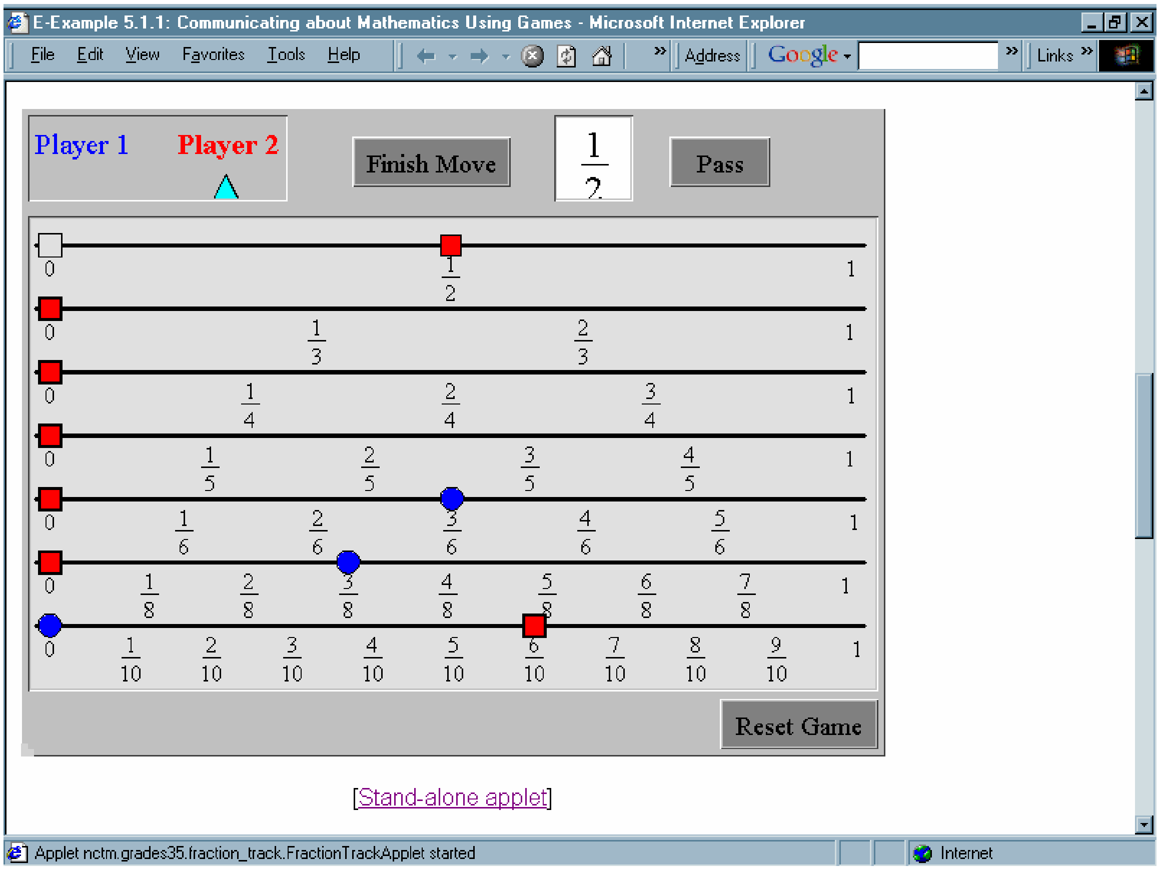
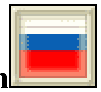


Figure 5. Task Sheet.

TASK 2: Two Step Sum 

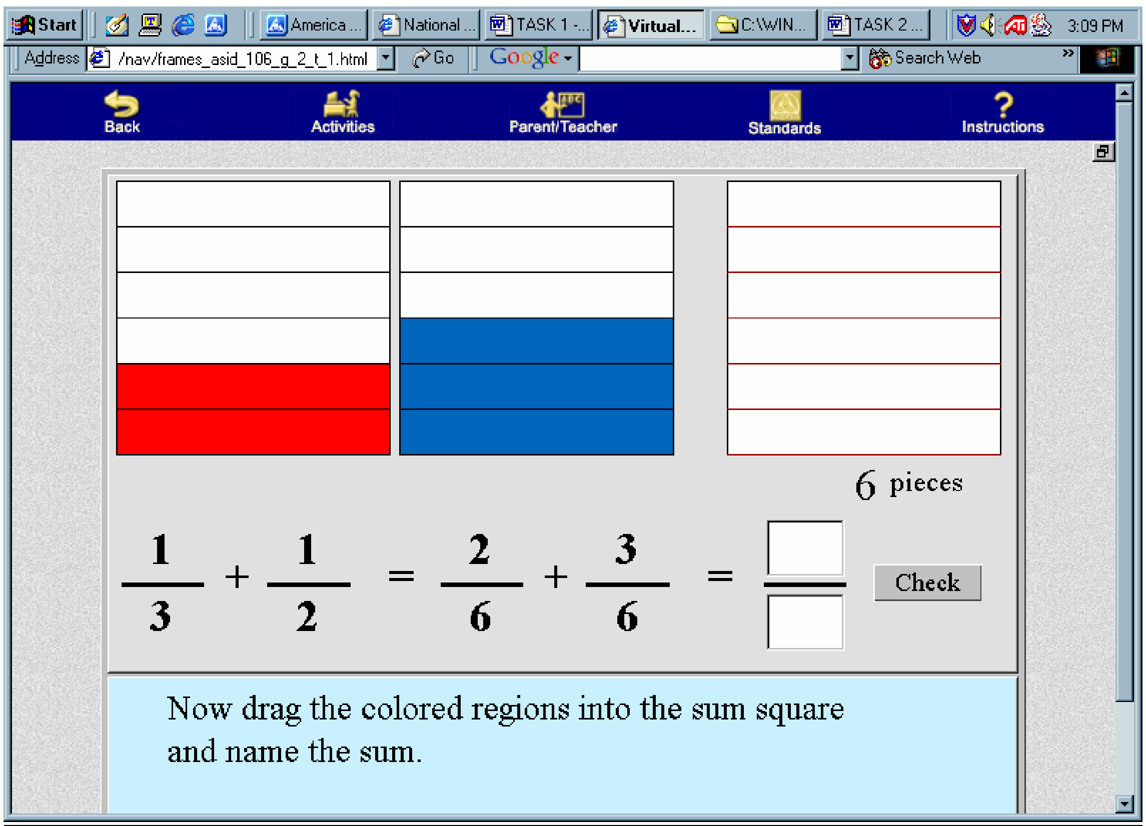
1. **Go to the NLVM at http://matti.usu.edu/nlvm/nav/vlibrary.html**

2. **Click on Number sense grade 3-5**

3. **Click on Addition of fractions.**

4. **Try 5 problems.**

5. **As you try the problems, record your work on the back of this sheet.**



This manipulative illustrates the two-step process of adding proper fractions. The first step in adding fractions is to identify a common group name (denominator). Finding a common group name means separating two (or more) whole groups into the same number of parts.

To complete this activity:

1. Use the arrow keys to separate the whole units into the same number of parts.
2. Enter the appropriate numerator and denominator values for the renamed fractions.
   Renamed fractions are equivalent. Can you state a rule for renaming?

   The second step is to combine (or add) the renamed fractions.
3. Drag the highlighted parts to form the new graph and type the resulting sum into the fraction box.
4. Click on the "Check" button to see if your answer is correct.

Keep a record of your work in the space below.

How would you state what you did in YOUR OWN words?

____
____
____
____
____
____
____

Plus	
Minus	
Interesting	

Go and play the Fraction Tracks game at
http://standards.nctm.org/document/eexamples/chap5/5.1/index.htm